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A Mathematical Technique For
Estimating True Temperature Profiles Of
Data Obtained From Long Time
Constant Thermocouples

Graeme David Young

DSTO-TN-0133

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DSTO-TN-0133

ABSTRACT

A mathematical modelling technique is described for estimating true temperature profiles of data obtained from long time constant thermocouples, which were used in fuel fire tests designed to determine the sensitivity of explosive ordnance. Although acquired temperature data indicated a test failure, the modelling and ensuing analysis showed that the test was valid. The devised model is then further investigated to provide a higher degree of confidence in the original methodology and conclusions.

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Executive Summary

This technical note documents original work conducted in establishing a mathematical modelling technique for estimating true temperature profiles of data obtained from long time constant thermocouples. It then describes further analysis of the original modelling and ensuing results, which was performed to provide a higher degree of confidence in the original methodology and conclusions.

Thermocouples were used to measure temperature in smoke generator tests, in which the criteria for a successful test was that at least two thermocouples, out of a total of four (placed in specified positions around the ordnance), measured a temperature above 550°C within 30 seconds of test ignition. These tests involve fuel fire testing to determine the sensitivity of explosive ordnance in accordance with International and Australian Insensitive Munitions policies. As the selected thermocouples had a time constant of approximately 13 seconds, the readings from the thermocouples did not accurately indicate the true temperature at a given time. Instead, the readings had a marked lag, and the test results were inconclusive.

A mathematical modelling technique was used to estimate true temperature profiles of the data obtained from the thermocouples, knowing only the data acquired by the thermocouples at given time intervals, and the thermocouple time constant. The basic mathematical analysis consisted of modelling the thermocouple using Laplace transforms. By fitting a polynomial expression to the time dependent thermocouple output data, it was possible to compute the temperature stimulus. Further analysis was then performed to complement the original analysis, by examining observed anomalies in the original results, and by using expanded domains for the polynomial data fits.

Results from the modelling showed that two out of the four thermocouples did actually measure true temperatures above 550°C within 30 seconds after ignition, thus indicating a valid test. Further practical testing in the field was therefore not required.

Although this investigation was directed at finding a particular solution for a defined customer problem, the devised mathematical methodology, could be used in any scenario, where the time constant of a transducer acquiring data is large when compared to the actual data acquisition time, provided the time constant is itself not a variable.

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1. Background and Introduction

This technical note documents original work conducted in establishing a mathematical modelling technique for estimating true temperature profiles of data obtained from long time constant thermocouples. It then describes further analysis of the original modelling and ensuing results, to provide a higher degree of confidence in the original methodology and conclusions.

As stated in the [American Society For Testing and Materials {ASTM} Special Technical Publication 470 1970 p 131], 'no instrument responds instantly to a change in its environment. Thus, in a region where temperature is changing, a thermocouple will not be at the temperature of its environment and hence cannot indicate the true temperature'. The [ASTM 1970 p 132] then states that 'it is common practice to characterise the response of a temperature sensor by a first order thermal time constant τ '.

Thermocouples having a long thermal time constant were used in fuel fire tests by the Army's Engineering Development Establishment (EDE) Environmental Test Facility (ETF) Salisbury (now part of the Army Technology and Engineering Agency). The ETF had been tasked to perform fuel fire testing, one of a series of tests required to determine the sensitivity of explosive ordnance in accordance with international and Australian Insensitive Munitions policies. Previous work in this regard had been led by Explosives Ordnance Division (EOD) of MRL, and the thermocouples normally used by EOD were also used by ETF in the mini-fuel fire tests of Generators Smoke Training CSS60-80.

Using the acquired thermocouple data, the specified successful testing criteria that 'at least two thermocouples out of a total of four, placed in specified positions around the ordnance, measured temperatures above 550°C within 30 seconds after test ignition', was not met. However, as the thermocouples used had a thermal time constant of approximately 13 seconds which is long when compared to the actual duration of data acquisition, the readings from the thermocouples did not accurately indicate the true temperature at a given time. Instead, the readings had a marked lag, and the test results were therefore inconclusive.

It was not practicable to repeat the tests with shorter time constant thermocouples, so an analysis of the results was attempted to compensate for the long time constant. The original task therefore consisted of estimating actual true temperature profiles, knowing only the original acquired thermocouple data from the testing, and the test authority specified 13 second time constant of the thermocouple. Both the acquired data and time constant were assumed to be valid throughout the modelling and analysis.

The original analysis, however, did not investigate expanded domains for the acquired thermocouple data fit, and did not investigate what appeared to be offset anomalies in the calculated results. Further result analysis was therefore conducted some time after the original work was completed. The purpose of this further analysis was to provide a greater degree of confidence in the methodology initially used, by examining the observed anomalies in the original results, and by further modelling the results with a greater variety of parameter selection and range. This analysis provided the stimulus for the publication of the entire work as a technical note, as the original work by the author had not been formerly published.

The original :

- method of approach,
- modelling procedure,
- implementation of the model and
- original conclusions

will first be examined,
followed by subsequent analysis in greater detail of :

- expanded domains for data fits and,
- offset anomalies in the original results at time $t = 0$ seconds.

2. Original Method Of Approach

After preliminary investigations into the problem, it was decided that a mathematical approach would be the best method available to compensate for the thermocouple time constant, as test re-runs using shorter time constant thermocouples were not feasible at the time. This eventually proved to be relatively straightforward using Laplace transforms, and is depicted in Figure 1 which shows the basic system as a function of time (t) and the equivalent system transformed into the S plane.

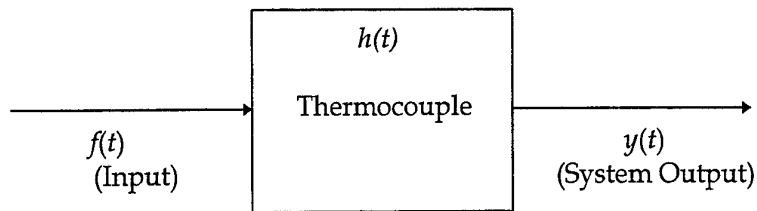
- $f(t)$ is the desired temperature stimulus which is required to be found,
- $y(t)$ is the measured temperature by the thermocouple, and
- $h(t)$ is the transfer function (output/input) of the thermocouple.

The thermocouple was modelled using Laplace transforms as $H(s)$, a polynomial expression was fitted to the time dependent thermocouple output data and transformed to $Y(s)$, the resultant input was calculated as $F(s) = Y(s)/H(s)$, and then the inverse Laplace transform was applied to predict the form of the temperature stimulus $f(t)$. The entire procedure therefore is basically a 'de-convolution' of $Y(s)$.

For the system in Figure 1 the output can be expressed as :

$$Y(s) = F(s) H(s) \quad \text{therefore} \quad F(s) = Y(s) / H(s)$$

The problem is to find transforms for the output $Y(s)$ and the thermocouple $H(s)$.



Transforming the above system into the S plane gives

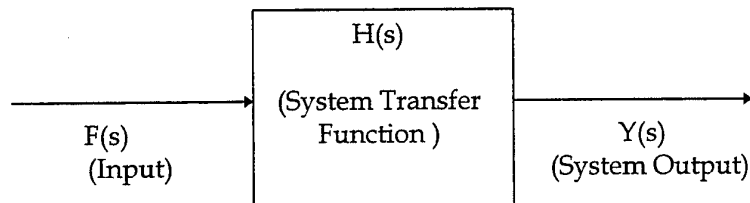


Figure 1: Mathematical Representations Of The System

3. Original Mathematical Modelling Procedure

3.1 Modelling the Output $y(t)$

Referring to Figure 1, $y(t)$ is the output produced by the thermocouple and is measured by the thermocouple during the fuel fire test. A least squares curve fitting program was used on this data to produce a polynomial expression for the output as a function of time, which was then easily transformed into the S plane as $Y(s)$.

Raw data obtained from thermocouples 1 & 4 is shown in Appendix A Table A.1, along with a plot in Figure A.1. Note that data for thermocouples 2 & 3 has not been used, as it was obvious from the acquired data that they would not have measured the required 550°C at any time, having only measured maximums less than 150°C , 30 seconds after ignition.

Each thermocouple data set was referenced to ambient temperature, and data from zero to 31 seconds was then input into a curve fitting program. Referencing to ambient was done as it was assumed that more than five time constants had elapsed before ignition, and it was not initially known if the ambient offset would affect the results. The final model therefore represents temperature referenced to zero degrees. Hence, all results, need to be adjusted by adding the ambient value. As discovered in the later analysis, this referencing to ambient was not required.

The curve fitting program used, was a modified GW Basic version of that described in [Rugg & Feldman 1980]. This program fits a calculated polynomial $y(t)$ to a set raw data pairs according to Equation (1).

$$y(t) = \sum_{k=0}^N a_k t^k \quad (1)$$

where

- t is time,
- N is the order of the polynomial, and
- a_k are the coefficients.

Increasing the polynomial order N , results in a better fit to the data points. This is measured by the percentage goodness of fit (PGF). The PGF is calculated as shown in Equation (2).

$$\text{PGF} = 100 \sqrt{1 - \frac{\sum_i (y_i - \hat{y}_i)^2}{\sum_i (y_i - \bar{y})^2}} \quad (2)$$

where

- y_i are the actual y raw data values,
- \hat{y}_i are the calculated y values using Equation (1),
- \bar{y} is the mean value of y , and
- i is the number of raw data pairs entered for the fit.

3.1.1 Data For Thermocouple One (TC #1)

TC #1 raw data was referenced to a measured ambient temperature of 17.6 C and a second order polynomial fit with a PGF of 99.6% is given in Equation (3). This model

was considered accurate enough for TC #1, as the ensuing analysis showed that this thermocouple had easily reached the required temperature with no ambiguity.

$$y_1(t) = 0.3t^2 + 4.93t - 5.82 \quad (3)$$

3.1.1 Data For Thermocouple Four (TC #4)

TC #4 raw data was referenced to a measured ambient temperature of 19.7 C^1 and third and fourth order polynomials yielding PGF's of 99.86% and 99.87% respectively are given in Equations (4 & 5).

$$y_4(t) = -0.025t^3 + 1.34t^2 - 5t + 10 \quad (4)$$

$$y_4(t) = -0.00084t^4 + 0.029t^3 + 0.237t^2 + 2.568t + 0.36 \quad (5)$$

Two polynomials were used to model this system output in order to investigate the dependence of results on the type of data fit, as the ensuing analysis showed a marginal test pass. Different degrees of polynomials will produce differing error profiles in the fit, and Figure (2) shows the actual error in degrees Celsius between the actual acquired raw thermocouple data, and the fitted data calculated from the third and fourth order polynomials given in Equations (4 & 5).

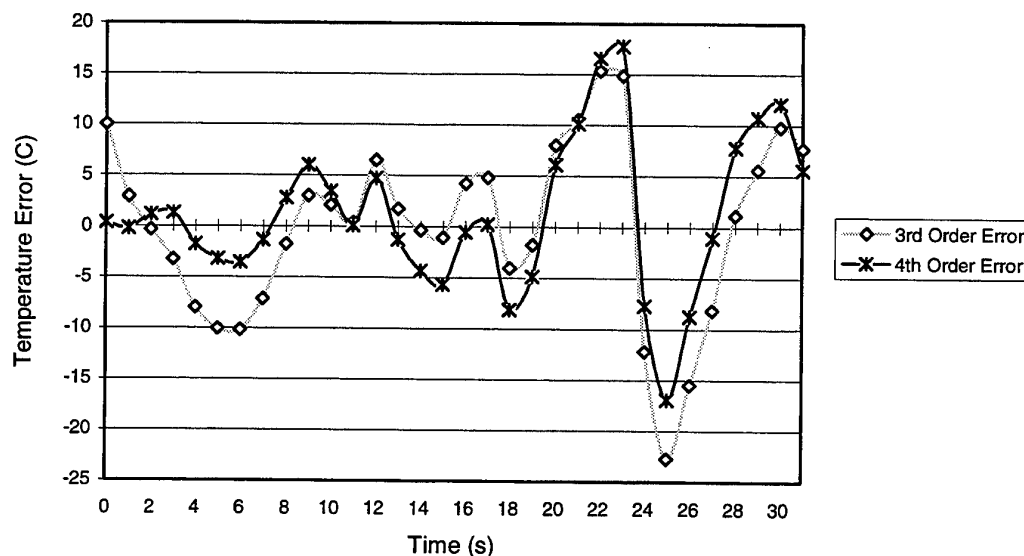


Figure 2: Actual Error In Degrees Celsius Between The Acquired Raw Data and Fitted Data From the Third and Fourth Order Polynomials For TC #4

¹ The difference in measured ambient for TC #1 and TC #4 was assumed to have resulted from an error in the thermocouple calibration at low temperatures.

3.2 Modelling the Thermocouple H(s)

To derive an appropriate mathematical model for the thermocouple, the following published information from the [ASTM 1970 P 131 - 134] was used. The [ASTM 1970 p 132] states that the

'thermocouple can be represented by a first order, first degree, linear, differential equation, the solution of which is given in Equation (6).

$$T = Ce^{-t/\tau} + \frac{1}{\tau} e^{-t/\tau} \int_0^t T_e e^{t'/\tau} dt' \quad (6)$$

where

- T is the sensor temperature at time t ,
- T_e is the environment temperature at time t ,
- C is a constant of integration, and
- τ is the time constant.

For a ramp change in temperature (as found in a furnace being heated at a uniform rate), Equation (6) reduces to that shown in Equation (7).

$$(T_e - T) = R\tau \quad (7)$$

Equation (7) states that if an element is immersed for a long time in an environment whose temperature is rising at a constant rate $R = dT_e/dt$, then τ is the interval between the time when the environment reaches a given temperature (T_e) and the time when the element reading (T) indicates this temperature.'

Figure 3 shows this graphically.

'For a step change in temperature (as when a thermocouple is plunged into a constant temperature bath), Equation (6) reduces to that shown in Equation (8).

$$(T_e - T) = (T_e - T_1) e^{-t/\tau} \quad (8)$$

where

- T_e is the final temperature value to be reached,
- T_1 is the initial temperature value, and
- T is the thermocouple element reading.

Equation (8) shows that for a step input, after one time constant (1τ) the sensor will indicate 63.2% of the temperature step value ($T_e - T_1$), and for practical purposes the sensor will reach the new temperature after approximately 5 time constants (5τ).

Figure 4 shows this graphically.

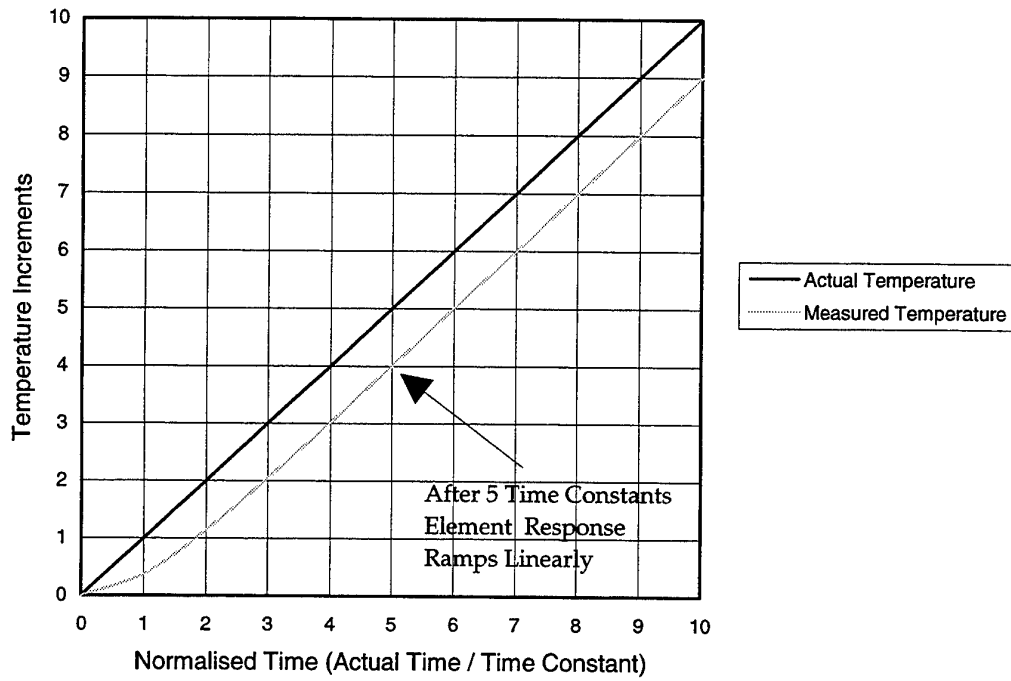


Figure 3: Thermocouple Response To Ramp Change

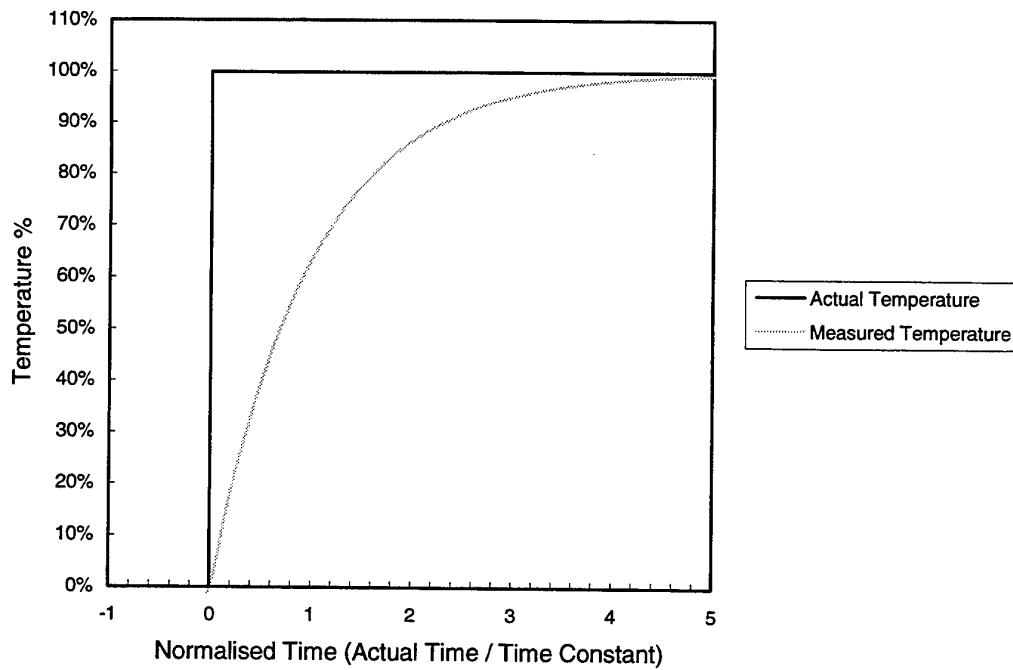


Figure 4: Thermocouple Response To Step Change

Note that the ordnance ignition could be thought of in terms of a step change followed by an unknown (but ramp like) increase in temperature. This is however an over simplification but is useful to show the effects of the thermocouple time constant.

Effects on the time constant value are also described in the [ASTM 1970 p 132,133], but for this particular purpose, it has been assumed that the time constant will remain a constant 13 seconds (i.e. $\tau = 13$), especially as it later stated in the [ASTM 1970 p 133], that 'the time constant usually is represented adequately by the first order time constant τ .'

From the foregoing discussion, it was originally concluded by the author, that the thermocouple could be modelled in electronic terms as an RC integrating network with a time constant of 13 seconds. This information is represented in Figure 5.

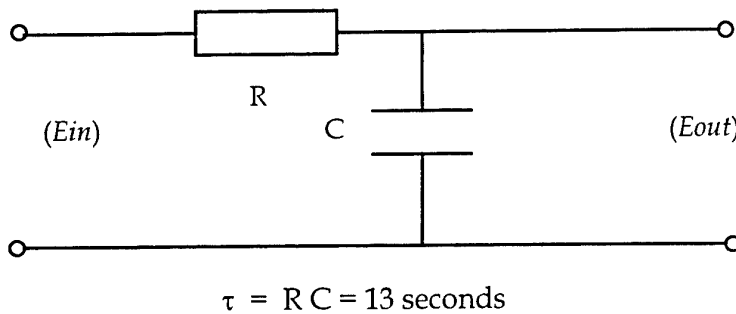


Figure 5: Electronic Model Of Thermocouple

The transfer function of the RC integrating circuit $H(s)$ (i.e. output (E_{out}) /input (E_{in})) is derived as shown in Equation (9) and becomes the mathematical model for the thermocouple, with RC (τ) assumed to be a constant 13 seconds.

$$H(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{RCs + 1} = \frac{1}{\tau s + 1} \quad (9)$$

Note that the method of obtaining $H(s)$ by using an electrical analogy for the thermocouple, was used by the author simply due to familiarity in this area of technology. A full derivation is given in [Lancaster 1987].

It was later shown by Mr. Martin Gill, a Senior Research Scientist of the Communications Division, Electronic and Surveillance Research Laboratory (ESRL) Defence Science and Technology Organisation Salisbury (DSTO), that the thermocouple transfer function could have been derived directly from Equation (6). The working for this is given in Appendix B.

4. Original Implementation Of Model

4.1 TC # 1 Implementation

Equation (3) represents the data acquired by TC #1. This is reproduced below and then transformed into the S plane as Equation (10) by using a table of selected Laplace transforms given in [Cochran *et al.* 1987]. The required system input $F_1(s)$ is then calculated in Equation (11).

$$y_1(t) = 0.3t^2 + 4.93t - 5.82$$

Taking the Laplace transform results in :

$$Y_1(s) = \frac{0.6}{s^3} + \frac{4.93}{s^2} - \frac{5.82}{s} \quad (10)$$

Applying the output to the system transfer function gives the input in the S plane as :

$$\begin{aligned} F_1(s) &= \frac{Y_1(s)}{H(s)} = (\tau s + 1) \left(\frac{0.6}{s^3} + \frac{4.93}{s^2} - \frac{5.82}{s} \right) \\ &= \frac{0.6\tau}{s^2} + \frac{0.6}{s^3} + \frac{4.93\tau}{s} + \frac{4.9}{s^2} - \left(5.8\tau + \frac{5.8}{s} \right) \end{aligned} \quad (11)$$

Note that in Equation (11), the 5.8τ is a constant term in the S plane. When transformed back to the time domain, this only produces an impulse response at time zero, and has therefore been ignored.

The inverse transform of Equation (11) (but ignoring the 5.8τ) term, yields Equation (12), which is the required true temperature polynomial.

$$\begin{aligned} f_1(t) &= 0.6\tau t + 0.3t^2 + 4.93\tau + 4.9t - 5.8 \\ &= 0.3t^2 + 12.7t + 58.3 \end{aligned} \quad (12)$$

At $t = 30$ and $\tau = 13$ seconds, Equation (12) yields 709°C . Adding the ambient yields 726°C , which is in excess of the required 550°C , thus indicating a test pass. A plot of results is shown in Figure 6.

Note that results are obtained assuming that the polynomial data fits for the acquired temperature output data $y(t)$ are valid from zero to infinity seconds (i.e. the integration period over which the Laplace transform for a function is obtained).

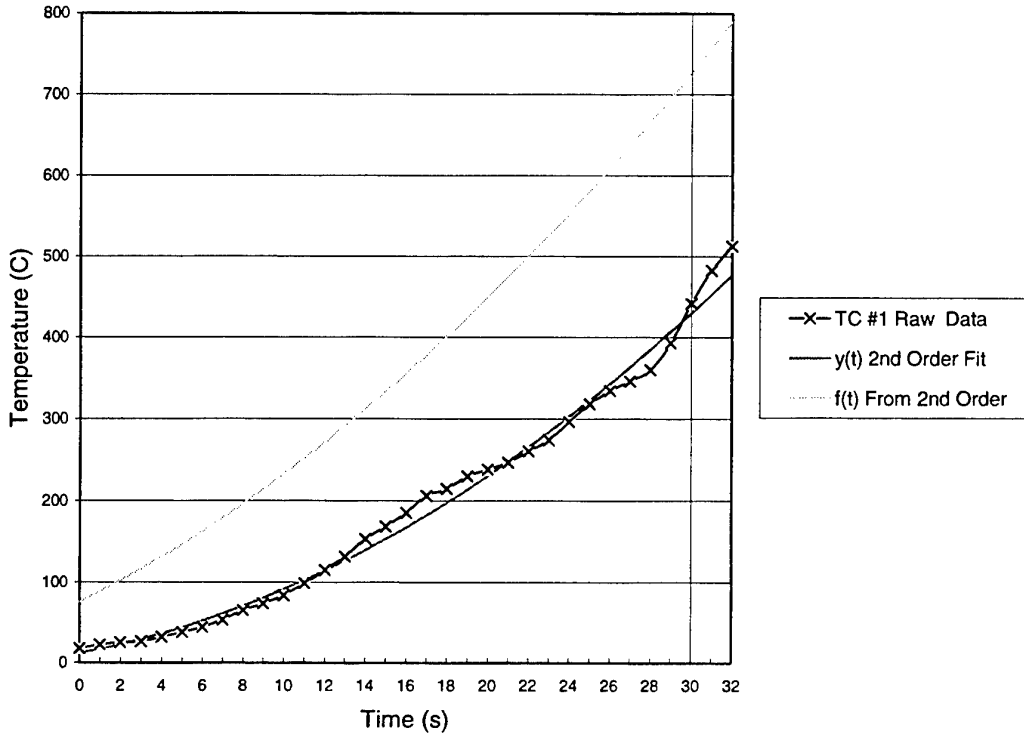


Figure 6: TC #1 Acquired Raw Data, Second Order Data Fit $y(t)$, and Estimated True Temperature $f(t)$ (All Plots Ambient Adjusted)

4.2 TC # 4 Implementation

Equations (4 & 5) represent two models of the data acquired by TC #4. The model from Equation (4) will be investigated first. It is reproduced below and then transformed into the S plane as Equation (13). The required system input $F_4(s)$ is then calculated in Equation (14) and transformed into the time domain in Equation (15).

$$y_4(t) = -0.025t^3 + 1.34t^2 - 5t + 10$$

Taking the Laplace transform results in :

$$Y_4(s) = -\frac{0.15}{s^4} + \frac{2.68}{s^3} - \frac{5.0}{s^2} + \frac{10}{s} \quad (13)$$

Applying the output to the system transfer function gives the input in the S plane as :

$$\begin{aligned}
 F_4(s) &= \frac{Y_4(s)}{H(s)} = (\tau s + 1) \left(-\frac{0.15}{s^4} + \frac{2.68}{s^3} - \frac{5.0}{s^2} + \frac{10}{s} \right) \\
 &= -\frac{0.15\tau}{s^3} - \frac{0.15}{s^4} + \frac{2.68\tau}{s^2} + \frac{2.68}{s^3} - \frac{5.0\tau}{s} - \frac{5.0}{s^2} + \frac{10}{s} + 10\tau
 \end{aligned} \quad (14)$$

Note that in Equation (14), the 10τ is a constant term in the S plane. As was done for TC #1, this is ignored in the results.

The inverse transform of Equation (14) (but ignoring the 10τ term), yields Equation (15), which is the required true temperature polynomial.

$$\begin{aligned}
 f_4(t) &= -0.075\tau t^2 - 0.025t^3 + 2.68\tau t + 1.34t^2 - 5.0\tau - 5t + 10 \\
 &= -0.025t^3 + 0.365t^2 + 29.84t - 55
 \end{aligned} \quad (15)$$

At $t = 30$ and $\tau = 13$ seconds, Equation (15) yields 494°C . Adding the ambient yields 514°C , which is below the required 550°C . However this is not the maximum of the polynomial. The derivative of Equation (15) is shown in Equation (16), of which the roots to the quadratic are 25.4 and -15.6. Therefore there is a turning point at 25.4. The second derivative of Equation (15) is shown in Equation (17) which is negative at $t = 25.4$, thus indicating a local maximum at $t = 25.4$.

$$\frac{df_4}{dt} = -0.075t^2 + 0.73t + 29.8 \quad (16)$$

$$\frac{d^2f_4}{dt^2} = -0.150t + 0.73 \quad (17)$$

At $t = 25.4$, Equation (15) yields 530°C . Adding the ambient yields 549.6°C , which is a very marginal result on the required 550°C .

The fourth order polynomial fit for TC #4 will now be investigated in a similar manner to that done for the third order.

Equation (5) is reproduced below and then transformed into the S plane as Equation (18).

$$\begin{aligned}
 y_4(t) &= -0.00084t^4 + 0.029t^3 + 0.237t^2 + 2.568t + 0.36 \\
 Y_4(s) &= -\frac{0.2}{s^5} + \frac{0.174}{s^4} + \frac{0.474}{s^3} + \frac{2.568}{s^2} + \frac{0.36}{s}
 \end{aligned} \quad (18)$$

$$\begin{aligned}
F_4(s) = \frac{Y_4(s)}{H_4(s)} &= (\tau s + 1) \left(-\frac{0.2}{s^5} + \frac{0.174}{s^4} + \frac{0.474}{s^3} + \frac{2.568}{s^2} + \frac{0.36}{s} \right) \\
&= -\frac{0.02\tau}{s^4} - \frac{0.02}{s^5} + \frac{0.174\tau}{s^3} + \frac{0.174}{s^4} + \frac{0.474\tau}{s^2} \\
&\quad + \frac{0.474}{s^3} + \frac{2.568\tau}{s} + \frac{2.568}{s^2} + \frac{0.36}{s} + 0.36\tau
\end{aligned} \tag{19}$$

The inverse transform of Equation (19) (but ignoring the 0.36τ term, yields Equation (20), which is the required true temperature polynomial.

$$f_4(t) = -0.00083t^4 - 0.0139t^3 + 1.361t^2 + 8.722t + 33.64 \tag{20}$$

At $t = 30$, Equation (20) yields 472°C . Adding the ambient yields 492°C , which is below the required 550°C . However, this is not the maximum of the polynomial. The maximum can be found using a similar procedure to that for Equation (15), in which a maximum turning point for a real root of the first derivative is found at $t = 24.85$. (The derivative of Equation (20) has three real roots of approximately 24.85, -34.4 and -3.1. (Note: The roots to the cubic polynomial derivative of Equation (20) were found using an unpublished 'C' program 'CubicRoot' by G. D. Young.)

At $t = 24.85$, Equation (20) yields 561°C . Adding the ambient yields 581°C , which is in excess of the required 550°C , thus indicating a test pass using the fourth order polynomial fit.

5. Original Analysis Of Model Results

5.1 TC # 1 Analysis

The results from section 4.1 indicate that even with a simple data fit for the output of TC #1, at $t = 30$, the temperature was calculated to be approximately 726°C , which is far in excess of the required 550°C . TC #1 therefore easily reached the required temperature at the specified time, and will not be dealt with in the remainder of this technical note.

5.2 TC #4 Analysis

From section 4.2, the results obtained with a third order polynomial data fit, represent only a marginal test pass, but with a fourth order polynomial fit, the estimated output of TC #4, at $t = 24.85$, was calculated to be approximately 581°C , which indicates an acceptable test pass. Figure 7 shows ambient adjusted plots, for TC #4 original acquired raw temperature data, against plots for the estimated actual true temperature $f_4(t)$, using both the third and fourth order polynomial data fits. The original results for TC #4 were marginal, but adequate for the purpose of the original exercise. Further investigation into these results however will be conducted in section 7 of this technical note to examine :

- the effect of the domain of acquired data used for the fit, and
- reasons for the observed offset occurring in the estimated results at $t = 0$.

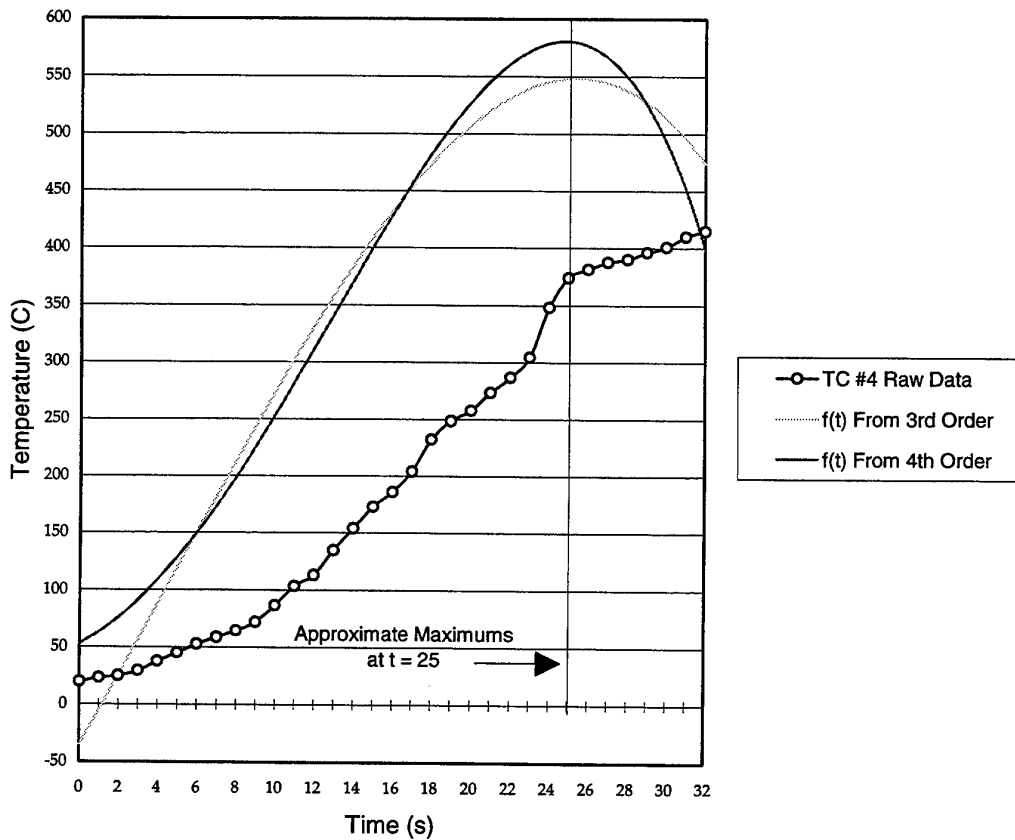


Figure 7: TC #4 Acquired Raw Data and Estimated True Temperature Data $f(t)$ For 3rd and 4th Order Data Fits (All Plots Ambient Adjusted)

6. Original Conclusions

Without further analysis, the following original conclusions were established :

Within the accuracy of the curve fitting procedure, acquired data, and thermocouple time constant factor, it can be concluded that :

- Two thermocouples have measured a true temperature in excess of 550°C within 30 seconds of ignition, but one must be considered marginal.
- The accuracy of the results is very dependent on the type of data fit used for the thermocouple acquired temperature data, and the results from the mathematical process can only be considered estimates.

7. Further Result Analysis

7.1 Investigation Of Expanded Domains For The Data Fits

It is evident from Figure 7, that the data fit for $y(t)$ has a significant impact on the estimated results obtained. It can therefore be assumed, that the time period over which the data is modelled will also have an impact on the results. To investigate this assumption, TC #4 data was modelled using a third order polynomial, over a domain from zero to 45 seconds after ignition. Equation (21) shows the model referenced to zero degrees. A data fit of 99.48% was obtained.

$$y_4(t) = -0.00947t^3 + 0.5415t^2 + 5.44t - 14.8 \quad (21)$$

Using the data fit in Equation (21) instead of Equation (4) in section 4.2, yields Equation (22) for $f_4(t)$, which is the required estimated true temperature polynomial referenced to zero degrees.

$$f_4(t) = -0.00946t^3 + 0.17t^2 + 19.48t + 55.90 \quad (22)$$

This result (adjusted for ambient) is plotted in Figure 8, and once again shows a marginal pass for TC #4, with an estimated temperature of 557.6°C at $t = 30$ after adding the ambient offset. It can therefore be concluded that when using an expanded domain for the third order polynomial data fit, TC #4 has measured a true temperature in excess of 550°C within 30 seconds of ignition, although the result must be considered marginal.

TC #4 data was also modelled using a fourth order polynomial, over a domain from zero to forty five seconds. Equation (23) shows the model referenced to zero degrees. A data fit of 99.81% was obtained.

$$y_4(t) = 0.000638t^4 - 0.06688t^3 + 2.19t^2 - 10.63t + 17.7 \quad (23)$$

Using the data fit in Equation (23) instead of Equation (5) in section 4.2, yields Equation (24) for $f_4(t)$, which is the required estimated true temperature polynomial referenced to zero degrees.

$$f_4(t) = 0.0006375t^4 - 0.0333t^3 - 0.41t^2 + 46.3t - 120 \quad (24)$$

Equation (24) (adjusted for ambient) is also shown plotted in Figure 8.

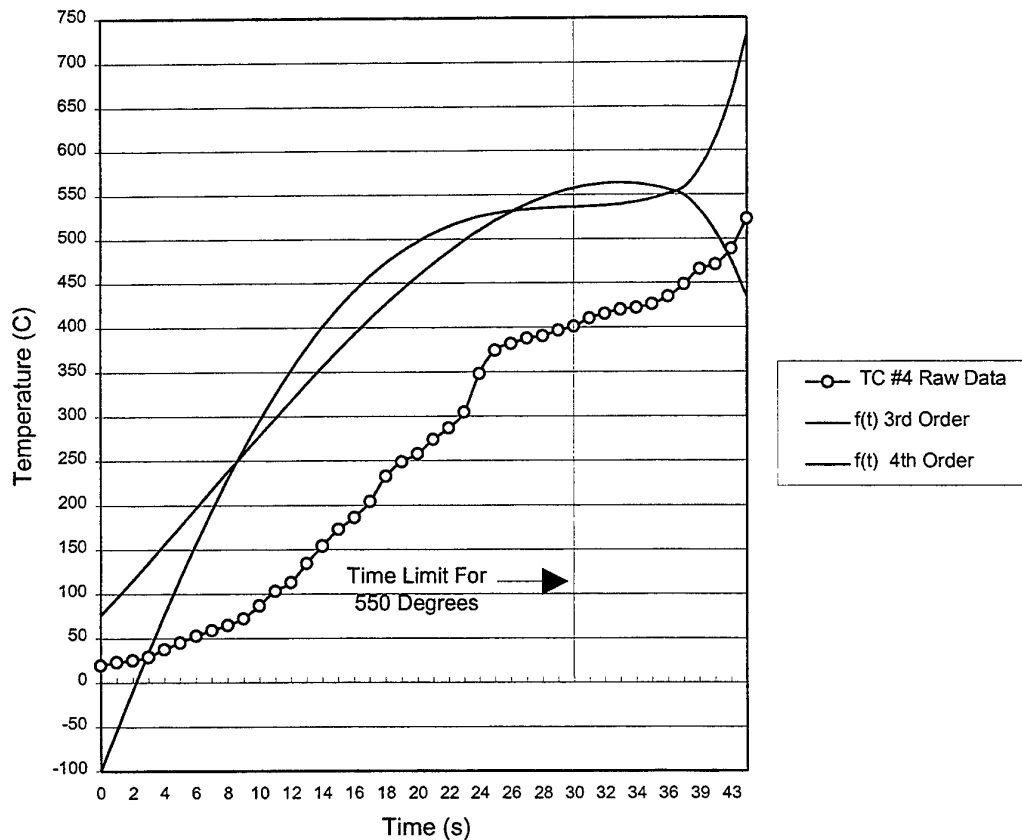


Figure 8: TC #4 Acquired Raw Data and Estimated True Temperature Data $f(t)$ For Expanded 3rd and 4th Order Data Fits (All Plots Ambient Adjusted)

The predicted results from Equation (24) however, show a marginal fail for TC #4, with an estimated temperature of 537.0°C at $t = 30$ after adding on the ambient.

The results from Equations (22 & 24), indicate that the type of data fit, and the time domain chosen for the data fit, have a significant effect on the final results, and concur with the original conclusions in section 6.

7.2 Investigation Of The Calculated Offset Anomaly

Of some concern in the original estimated results, was the large temperature offsets at $t = 0$ for both the third and fourth order results, even though the model was derived with zero temperature offset. However, as the results around $t = 0$ have no bearing on the test pass/fail criteria, this anomaly can be partially ignored. The reason for the large offsets at $t = 0$, can be explained by investigating the first order t term in each of the fitted polynomials. This term represents a pure ramp output, and as explained in the following text, is the predominate term in producing the large $t = 0$ offsets.

Assume the system transfer function is identical to the 13 second time constant thermocouple (shown in Equation (25)).

$$\text{i.e.} \quad H(s) = \frac{1}{13s + 1} \quad (25)$$

Assume also that the system output is a linear ramp as shown in Equation (26).

$$\text{i.e.} \quad y(t) = t \quad (26)$$

Equation (26) transformed into the S plane is given in Equation (27).

$$Y(s) = \frac{1}{s^2} \quad (27)$$

The estimated system input $F(s)$ is shown in Equation (28).

$$F(s) = \frac{Y(s)}{H(s)} = \frac{13s + 1}{s^2} = \frac{13s}{s^2} + \frac{1}{s^2} = \frac{13}{s} + \frac{1}{s^2} \quad (28)$$

The inverse transform for $F(s)$ in Equation (28), is given in Equation (29).

$$f(t) = 13 + t \quad (29)$$

Equation (29) indicates the input must consist of a ramp identical to the output but offset by a step of magnitude 13 at $t = 0$. This behaviour is in agreement with that explained in section 3.2 and in particular Figure 3.

8. Final Conclusions

This technical note has shown a mathematical modelling technique for estimating true temperature profiles of data obtained from long time constant thermocouples which were used in fuel fire tests to determine the sensitivity of explosive ordnance. The thermocouple was modelled using Laplace transforms as $H(s)$, a polynomial expression was fitted to the time dependent thermocouple output data and transformed to $Y(s)$, the resultant input was calculated as $F(s) = Y(s)/H(s)$, and then the inverse Laplace transform was applied to predict the form of the temperature stimulus $f(t)$.

Within the accuracy of the curve fitting procedure, acquired data, and thermocouple time constant factor, it can be concluded that :

- the mathematical modelling has shown that two thermocouples have measured a true temperature in excess of 550°C within 30 seconds of ignition, but one must be considered marginal. This indicates a valid test, whereas the acquired raw temperature data indicated a test failure.
- The accuracy of the results is very dependent on the type of data fit and time domain used for the thermocouple acquired temperature data, and the results from the mathematical process can only be considered estimates.
- The mathematical modelling method of approach appears reasonable.

9. References

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Appendix A

Data Acquisition Time (seconds) After Ignition	TC # 1 Acquired Temperature Data	TC # 4 Acquired Temperature Data
0	17.6	19.7
1	22.4	23.1
2	25.2	25.2
3	26.5	29.3
4	32.0	37.5
5	38.2	45.1
6	44.4	52.7
7	53.4	58.9
9	73.4	72.0
10	83.8	86.6
11	98.4	103.2
12	115.0	113.0
13	131.8	134.5
14	153.4	154.1
15	168.7	172.9
16	185.4	186.1
17	206.3	204.3
18	214.7	232.1
19	230.0	248.7
20	238.3	257.7
21	246.6	273.6
22	260.5	286.7
23	274.3	304.5
24	296.3	348.1
25	318.2	374.3
26	334.5	381.6
27	346.1	387.6
28	360.2	390.3
29	392.9	396.3
30	441.7	400.9
31	482.7	410.0
32	512.5	415.3
33	523.7	419.9
34	531.7	421.9
35	546.2	425.9
37	594.6	448.3
39	619.9	465.5
41	629.9	470.8
43	641.3	488.7
45	626.6	522.4

Table A.1: Sample Of Acquired Raw Data For TC #1 and TC #4

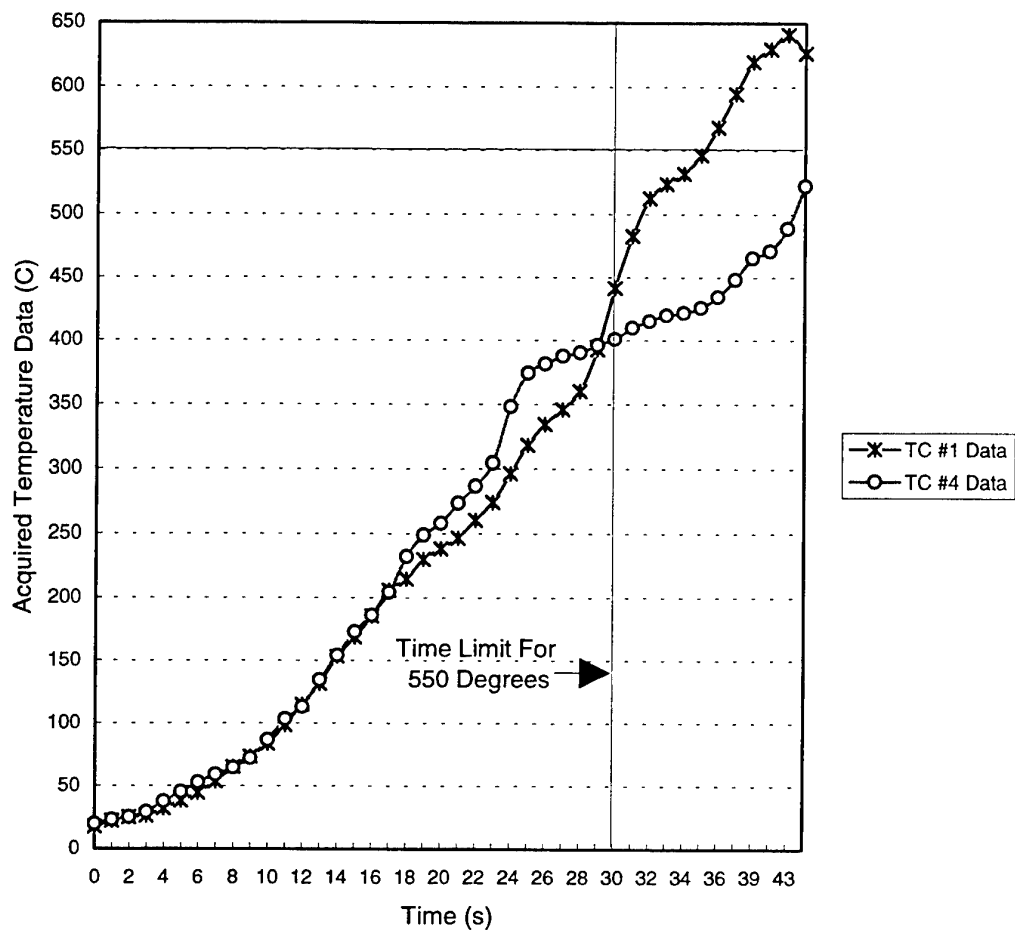


Figure A.1: Acquired Raw Data For TC #1 and TC #4

Appendix B

The following derivation of the thermocouple transfer function from its descriptive equation, was performed by Mr. Martin Gill, a Senior Research Scientist of the Communications Division, Electronic and Surveillance Research Laboratory (ESRL) Defence Science and Technology Organisation Salisbury (DSTO).

As stated in the body of the technical note, the thermocouple can be represented by a first order, first degree, linear, differential equation, the solution of which is given in Equation (B.1).

$$T(t) = Ce^{-t/\tau} + \frac{1}{\tau} e^{-t/\tau} \int_0^t T_e(t) e^{t'/\tau} dt \quad (\text{B.1})$$

where

- T is the sensor temperature at time t ,
- T_e is the environment temperature at time t ,
- C is a constant of integration, and
- τ is the time constant.

Differentiating Equation (B.1) with respect to t yields Equation (B.2) :

$$\frac{dT(t)}{dt} = \frac{-Ce^{-t/\tau}}{\tau} - \frac{1}{\tau} \left(\frac{1}{\tau} e^{-t/\tau} \int_0^t T_e(t) e^{t'/\tau} dt \right) + \frac{1}{\tau} e^{-t/\tau} T_e(t) e^{t'/\tau} \quad (\text{B.2})$$

Re-arranging Equation (B.1) we note that :

$$\frac{1}{\tau} e^{-t/\tau} \int_0^t T_e(t) e^{t'/\tau} dt = T(t) - Ce^{-t/\tau} \quad (\text{B.1}')$$

Substitution of (B.1') into (B.2) yields Equation (B.3) :

$$\begin{aligned} \frac{dT(t)}{dt} &= \frac{-Ce^{-t/\tau}}{\tau} - \frac{1}{\tau} (T(t) - Ce^{-t/\tau}) + \frac{1}{\tau} e^{-t/\tau} T_e(t) e^{t'/\tau} \\ &= \frac{1}{\tau} (T_e(t) - T(t)) \end{aligned} \quad (\text{B.3})$$

Transforming Equation (B.3) into the S plane but ignoring initial conditions yields Equation (B.4) :

$$sT(s) = \frac{1}{\tau} (T_e(s) - T(s)) \quad (\text{B.4})$$

Re-arranging Equation (B.4) yields Equation (B.5) :

$$T(s) \left(s + \frac{1}{\tau} \right) = \frac{T_e(s)}{\tau} \quad (\text{B.5})$$

The thermocouple output is $T(s)$, and the input is $T_e(s)$ and hence the transfer function $H(s)$ (output / input) is shown in Equation (B.6) :

$$H(s) = \frac{T(s)}{T_e(s)} = \left(\frac{1}{\tau} \right) \left(\frac{1}{s + \frac{1}{\tau}} \right) = \frac{1}{\tau s + 1} \quad (\text{B.6})$$

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